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Power Shaping: A New Paradigm for Stabilization of Nonlinear RLC Circuits

R. Ortega, D. Jeltsema, and J. M. A. Scherpen

Abstract—It is well known that arbitrary interconnections of passive (possibly nonlinear) resistors, inductors, and capacitors define passive systems, with port variables the external source voltages and currents, and storage function the total stored energy. In this note, we prove that for a class of RLC circuits with convex energy function and weak electromagnetic coupling it is possible to “add a differentiation” to the port terminals preserving passivity—with a new storage function that is directly related to the circuit power. The result is of interest in circuits theory, but also has applications in control as it suggests the paradigm of power shaping stabilization as an alternative to the well-known method of energy shaping. We show in this note that, in contrast with energy shaping designs, power shaping is not restricted to systems without pervasive dissipation and naturally allows to add “derivative” actions in the control. These important features, that stymie the applicability of energy shaping control, make power shaping very practically appealing. To establish our results we exploit the geometric property that voltages and currents in RLC circuits live in orthogonal spaces, i.e., Tellegen’s theorem, and heavily rely on the seminal paper of Brayton and Moser in 1964.

Index Terms—Nonlinear control, passivity, stability theory.

I. INTRODUCTION

In this note, we are interested in (possibly nonlinear) RLC circuits consisting of arbitrary interconnections of resistors, inductors, capacitors and voltage and current sources. It is well known that, if the resistors, inductors, and capacitors are passive, i.e., if their energy functions are positive, then the overall interconnected circuit is also passive with port variables the external sources voltages and currents, and storage function the total stored energy [3]. This property was exploited by Youla in 1959 [15], who proved that terminating the port variables of a passive RLC circuit with a passive resistor would ensure that “finite energy inputs will be mapped into finite energy outputs,” what in modern parlance says that injecting damping to a passive system ensures \mathcal{L}_2 -stability. Passivity can also be used to stabilize a nonzero equilibrium point, but in this case we must modify the storage function to assign a minimum at this point. If the storage function is the total energy we refer to this step as energy shaping, which combined with damping injection constitute the two main stages of passivity-based control (PBC) [9]. As explained in [10] and [14], there are several ways to achieve energy shaping, the most physically appealing being the so-called energy balancing PBC (or control by interconnection) method. With this procedure the storage function assigned to the closed-loop passive map is the difference between the total energy of the system and the energy supplied by the controller, hence, the name energy balancing. Unfortunately, energy balancing PBC is stymied by the presence of pervasive dissipation, that is, the existence of resistive elements whose power does not vanish at the desired equilibrium point. Another practical drawback of energy-shaping control is the limited ability to “speed up” the transient response. Indeed, as tuning in this kind of controllers is essentially

restricted to the damping injection gain, the transients may turn out to be somehow sluggish, and the overall performance level below par; see [13] for some representative examples.

Our main contribution in this note is the establishment of a new passivity property for a class of RLC circuits that provides the basis for a novel PBC design methodology that does not suffer from the two aforementioned drawbacks. To define the class, we assume that the energy functions of the inductors and capacitors are not just positive but actually *convex*, and that the electromagnetic coupling between the dynamic elements is weak. Henceforth, for the case of RC or RL circuits the latter condition is conspicuous by its absence [7].

The new passivity property, which is by itself of interest in circuits theory, has two key features that makes it attractive for control design as well. First, that the storage function is not the total energy, but a function directly related with the *power* in the circuit. Second, that the port variables of the new passive system include *derivatives* of the sources voltages and/or currents. The utilization of power (instead of energy) storage functions immediately suggests the paradigm of power shaping stabilization as an alternative to the well-known method of energy shaping. We show in the note that, in contrast with energy shaping designs, power shaping is applicable also to systems with pervasive dissipation, the only restriction for stabilization being the degree of underactuation of the circuit. Further, establishing passivity with respect to “differentiated” port variables allows the direct incorporation of (approximate) derivative actions, whose predictive nature can speed-up the transient response.

II. ENERGY BALANCING CONTROL AND A MOTIVATING EXAMPLE

In [11], we presented a new method to stabilize the following class of nonlinear systems.

Definition 1: We say that the m -port system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$, $\mathbf{y} = \hat{\mathbf{y}}(\mathbf{x})$, with state $\mathbf{x} = \text{col}(x_1, \dots, x_n) \in \mathbb{R}^n$, and power port variables¹ $\mathbf{u}, \mathbf{y} \in \mathbb{R}^m$, satisfies the energy balance inequality if, along all trajectories compatible with $\mathbf{u} : [0, t] \rightarrow \mathbb{R}^m$, we have

$$\underbrace{\mathcal{E}[\mathbf{x}(t)] - \mathcal{E}[\mathbf{x}(0)]}_{\text{stored energy}} \leq \underbrace{\int_0^t \mathbf{u}^\top(s) \hat{\mathbf{y}}[\mathbf{x}(s)] ds}_{\text{supplied}} \quad (1)$$

where $\mathcal{E} : \mathbb{R}^n \rightarrow \mathbb{R}$ is the stored energy function. If $\mathcal{E}(\mathbf{x}) \geq 0$ then we say that the system is passive with port variables (\mathbf{u}, \mathbf{y}) .

The proposition that follows constitutes the basis for energy-balancing PBC. (For simplicity, we present only the case of static state feedback, the dynamic case—also called control by interconnection—may be found in [11] and [14].)

Proposition 1: Consider m -port systems that satisfy the energy balance (1). If we can find a vector function $\hat{\mathbf{u}} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ such that the partial differential equation²

$$\nabla \mathcal{E}_a^\top [\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\hat{\mathbf{u}}(\mathbf{x})] = -\hat{\mathbf{u}}^\top(\mathbf{x})\hat{\mathbf{y}}(\mathbf{x}) \quad (2)$$

can be solved for the scalar function $\mathcal{E}_a : \mathbb{R}^n \rightarrow \mathbb{R}$, and the function $\mathcal{E}_a(\mathbf{x}) := \mathcal{E}(\mathbf{x}) + \mathcal{E}_a(\mathbf{x})$ has an isolated minimum at \mathbf{x}^* , then the state-feedback $\mathbf{u} = \hat{\mathbf{u}}(\mathbf{x})$ is an energy balancing PBC, i.e., \mathbf{x}^* is a

¹The variables \mathbf{u} and \mathbf{y} are assumed conjugate, in the sense that their product $\mathbf{u}^\top \mathbf{y}$ has units of power. For instance, voltages and currents or forces and velocities.

²We use the notation $\nabla_x := \partial/\partial x$, $\nabla_x^2 := \partial^2/\partial x^2$ —when clear from the context the argument will be omitted. Also, all vectors, including the gradient, are column vectors.

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stable equilibrium of the closed-loop with Lyapunov function $\mathcal{E}_d(\mathbf{x})$ that satisfies

$$\mathcal{E}_d[\mathbf{x}(t)] = \mathcal{E}[\mathbf{x}(t)] - \int_0^t \hat{\mathbf{u}}^\top[s] \hat{\mathbf{y}}[\mathbf{x}(s)] ds$$

thus, it equals the difference between the stored and the supplied energies.

It is shown in [11] that, beyond the realm of mechanical systems, the applicability of energy balancing control is severely stymied by the system's natural dissipation. Indeed, it is easy to see that a necessary condition for the *global* solvability of the PDE (2) is that $\hat{\mathbf{y}}^\top(\mathbf{x})\hat{\mathbf{u}}(\mathbf{x})$ vanishes at all the zeros of $\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\hat{\mathbf{u}}(\mathbf{x})$. Now, $\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\hat{\mathbf{u}}(\mathbf{x})$ is obviously zero at the equilibrium \mathbf{x}^* , hence, the power extracted from the controller should also be zero at the equilibrium. This means that energy balancing PBC is applicable only if the system does not have pervasive damping, i.e., if it can be stabilized extracting a finite amount of energy from the controller.

Let us illustrate with an example how the limitations of energy balancing PBC can be overcome via power balancing. Consider a voltage-controlled nonlinear series RL circuit. The behavior of the inductor is characterized by a function, $p_L = \hat{p}_L(i_L)$, relating the flux linkages p_L and the current i_L , and Faraday's law: $\dot{p}_L = v_L$, where v_L is the inductor voltage. The resistor is a static element described by its characteristic function $v_R = \hat{v}_R(i_R)$, where v_R, i_R are the resistors voltage and current, respectively. The dynamics of the circuit is obtained from Kirchhoff's voltage law as

$$v_L = L(i_L) \dot{i}_L = -\hat{v}_R(i_L) + v_S \quad (3)$$

where v_S is the voltage at the port terminal, which is our control action, we used $i_R = i_L$, and defined $L(i_L) := \nabla \hat{p}_L$. The energy stored in an inductor, $\mathcal{E}_L(p_L)$, is related with the current via the relation $i_L = \nabla \mathcal{E}_L$. Of course, if the resistor and the inductor are passive, the circuit defines a passive system with port variables (v_S, i_S) and storage function $\mathcal{E}_L(p_L)$.

We define as control objective the stabilization of an equilibrium i_L^* of (3), whose corresponding equilibrium supply voltage is given by $v_S^* = \hat{v}_R(i_L^*)$. If we further assume that the function $\hat{v}_R(i_R)$ is zero only at zero, it is clear that, at any equilibrium $i_L^* \neq 0$, the extracted power $i_L^* \hat{v}_R(i_L^*)$ is nonzero, hence, the circuit is not energy-balancing stabilizable—not even in the linear case. To overcome this problem let us define the function $G(i_R) := \int_0^{i_R} \hat{v}_R(i'_R) di'_R$, known in the circuits literature [12] as the resistors *content*, which has units of power—in particular, for linear resistors, where $\hat{v}_R(i_R) = Ri_R$, $R \in \mathbb{R}$, it is half the dissipated power. Furthermore, notice that for passive resistors the function is nonnegative and nondecreasing.

Proposition 2: Consider a series RL circuit. If the inductor is passive and has a twice differentiable convex energy function, then along the trajectories of the system, we have the power balance inequality³

$$G[i_L(t)] - G[i_L(0)] \leq \int_0^t v_S^\top(s) \hat{i}_S(s) ds. \quad (4)$$

Furthermore, if the resistor is passive, then the circuit is passive with port variables (v_S, \hat{i}_S) and storage function the resistor content.

Proof: Differentiating the resistors content with respect to time, we get

$$\dot{G} = v_R \hat{i}_L = (-v_L + v_S) \hat{i}_L = -\nabla^2 \mathcal{E}_L v_L^2 + v_S \hat{i}_S \leq v_S \hat{i}_S$$

³The name stems, of course, from the analogy with the energy balance inequality (1). A more accurate denomination is *resistors content* inequality, however, we will use the former for ease of reference.

where we have used (3) to get the second identity, taken the time derivative of $i_L = \nabla \mathcal{E}_L$ to get the third one and used convexity for the inequality. Integrating from 0 to t establishes (4), while the passivity property follows invoking nonnegativity of the content for passive resistors. \square

The properties of Proposition 2 differ from the classical energy-balancing and passivity properties in two important respects: the presence of the derivative of i_S and the use of a new power-like storage function. These two properties suggest, similarly to energy balancing PBC, to *shape the resistors content*. That is, to look for functions $\hat{v}_S(i_L)$, $G_a(i_L)$ such that

$$\dot{G}_a \equiv -\hat{v}_S(i_L) \hat{i}_L. \quad (5)$$

If we furthermore ensure that $i_L^* = \arg \min \{G(i_L) + G_a(i_L)\}$, then i_L^* will be a stable equilibrium with Lyapunov function $G(i_L) + G_a(i_L)$, that is, the system is stabilized via power shaping.

Clearly, for any choice of $G_a(i_L)$, (5) is trivially solved with the control $v_S = \hat{v}_S(i_L) = -\nabla G_a$. If the resistance characteristic is exactly known we can take $G_a(i_L) = -G(i_L) + (R_a/2)(i_L - i_L^*)^2$, with $R_a > 0$ some tuning parameter. However, to assign the desired minimum, we obviously only need to “dominate” $G(i_L)$ which (together with the fact that $L(i_L)$ is completely unknown) illustrates the robustness of the design procedure.

Remark 1: An important observation, that will be proved for more general nonlinear RLC circuits later, is that we can express the circuit dynamics (3) in terms of the resistor content as $L(i_L) \dot{i}_L = -\nabla G + v_S$. The identification of a gradient-like description of (a class of) RLC circuits is the main contribution of [2].

III. TELLEGEN'S THEOREM AND BRANCH BEHAVIOR

Tellegen's theorem is a fundamental result of general electrical networks that plays a central role in our developments and may be stated as follows [3].⁴

Proposition 3: (Tellegen's theorem) Consider an arbitrary lumped network whose graph has b branches and N nodes. Suppose that to each branch we assign arbitrarily a branch voltage v_k and a branch current i_k for $k = 1, \dots, b$. If these voltages and currents satisfy the constraints imposed by Kirchhoff's voltage and current laws, then $\mathbf{v}^\top \mathbf{i} = 0$, where we have defined $\mathbf{i} := \text{col}(i_1, \dots, i_b)$ and $\mathbf{v} := \text{col}(v_1, \dots, v_b)$.

The following remarks are in order.

- Since $v_k(t)i_k(t)$ is the power delivered at time t by the network to branch k , the theorem may be interpreted as the following conservation of energy statement: at any time t the sum of the power delivered to each branch of the network is zero.
- It is of crucial importance to realize that \mathbf{i} and \mathbf{v} are picked *arbitrarily*, subject only to Kirchhoff's laws. Consequently, the theorem has some rather astonishing consequences. For instance, if we consider two arbitrary lumped networks whose only constraint is to have the same graph, and denote (\mathbf{i}, \mathbf{v}) and $(\hat{\mathbf{i}}, \hat{\mathbf{v}})$ their corresponding branch currents and voltages, Tellegen's theorem guarantees that $\mathbf{v}^\top \hat{\mathbf{i}} = 0$ (and also $\hat{\mathbf{i}}^\top \mathbf{v} = 0$). Note that these expressions do not have an energy interpretation, because they involve voltages of one network and currents of another.

⁴We refer the interested reader to the classical references [2] and [3] for further details on circuit theory. See also [1], and [4]–[6] for material closely related with our developments.

Since Kirchhoff's voltage and current laws impose algebraic constraints, we have the following important corollary of Tellegen's theorem.

Corollary 1: Voltages and currents of an arbitrary lumped network satisfy

$$\mathbf{i}^\top \frac{d\mathbf{v}}{dt} = 0, \quad \mathbf{v}^\top \frac{d\mathbf{i}}{dt} = 0. \quad (6)$$

In this note, we consider RLC circuits consisting of interconnections of (possibly nonlinear) lumped dynamic (inductors, capacitors) and static (resistors and voltage and current sources) elements. We proceed now to define the behavior of the branch elements. An n_L -port inductor is defined by a vector function $\mathbf{p}_L = \hat{\mathbf{p}}_L(\mathbf{i}_L)$, with $\hat{\mathbf{p}}_L : \mathbb{R}^{n_L} \rightarrow \mathbb{R}^{n_L}$, and Faraday's law

$$\mathbf{v}_L = \dot{\mathbf{p}}_L = \mathbf{L}(\mathbf{i}_L) \frac{d\mathbf{i}_L}{dt} \quad (7)$$

where we defined the inductance matrix $\mathbf{L}(\mathbf{i}_L) := \nabla \hat{\mathbf{p}}_L$. Analogously, for n_C -port capacitors we have that the charges are related to the voltages as $\mathbf{q}_C = \hat{\mathbf{q}}_C(\mathbf{v}_C)$, with $\hat{\mathbf{q}}_C : \mathbb{R}^{n_C} \rightarrow \mathbb{R}^{n_C}$, and

$$\mathbf{i}_C = \dot{\mathbf{q}}_C = \mathbf{C}(\mathbf{v}_C) \frac{d\mathbf{v}_C}{dt} \quad (8)$$

where $\mathbf{C}(\mathbf{v}_C) := \nabla \hat{\mathbf{q}}_C$. We also have the following relationships for the energy functions $\mathcal{E}_L(\mathbf{p}_L)$, $\mathcal{E}_C(\mathbf{q}_C)$, where $\mathcal{E}_L : \mathbb{R}^{n_L} \rightarrow \mathbb{R}$, $\mathcal{E}_C : \mathbb{R}^{n_C} \rightarrow \mathbb{R}$,

$$\mathbf{i}_L = \nabla \mathcal{E}_L, \quad \mathbf{v}_C = \nabla \mathcal{E}_C. \quad (9)$$

In the sequel, we will assume that the energy functions are *twice differentiable*.

The circuit has n_R resistors, which are 1-ports characterized by a graph $v_{kR} = \hat{v}_{kR}(i_{kR})$, $k = 1, \dots, n_R$, where $\hat{v}_{kR} : \mathbb{R} \rightarrow \mathbb{R}$. (As explained later, we will sometimes find useful to use instead the graph $i_{kR} = \hat{i}_{kR}(v_{kR})$). It is clear that *constant* voltage and current sources can be easily added as particular instances of resistors. The network also contains *regulated* sources—that will interconnect the circuit with the controller. We denote their voltages and currents as $\mathbf{v}_S, \mathbf{i}_S \in \mathbb{R}^{n_S}$, respectively. In the sequel we will restrict our attention to regulated voltage sources. (See the discussion in Section VII for the case of current sources).

To simplify the notation, we will group all capacitors of the circuit into one n_C -port and all inductors into one n_L -port with corresponding energies the sum of the energies of all multi-port capacitors and inductors, respectively. Also, we will group all port variables into vectors denoted by $\mathbf{v} := \text{col}(\mathbf{v}_C, \mathbf{v}_L, \mathbf{v}_R, \mathbf{v}_S)$, $\mathbf{i} := \text{col}(\mathbf{i}_C, \mathbf{i}_L, \mathbf{i}_R, -\mathbf{i}_S)$, where we have adopted the standard sign convention for the sources currents.

IV. NEW PASSIVITY PROPERTY FOR RL AND RC CIRCUITS

In the sequel, we will assume that the circuit is *complete*, which means that the currents in the inductors and the voltages in the capacitors, via Kirchhoff's laws and the laws of the resistors characteristics, determine the voltages and currents in all the branches. Complete RLC circuits can be split into two subnetworks Σ_L, Σ_C that, respectively, contain all the inductors and capacitors; see [2]. According to this partition, we will split the resistors into two sets, the voltage-controlled resistors belonging to Σ_C , whose port variables will be denoted $(\mathbf{i}_{RC}, \mathbf{v}_{RC})$, and have characteristic functions $i_{kRC} = \hat{i}_{kRC}(v_{kRC})$; and the current-controlled resistors belonging to Σ_L , with port variables $(\mathbf{i}_{RL}, \mathbf{v}_{RL})$ and characteristic functions $v_{kRL} = \hat{v}_{kRL}(i_{kRL})$.

We now define the concepts of *content* and *co-content* of a resistor, which are well known in circuit theory [12], and will be instrumental to formulate our results.

Definition 2: The co-content of a voltage-controlled resistor and the content of current-controlled resistor are, respectively, defined as

$$J_k(v_{kRC}) := \int_0^{v_{kRC}} \hat{i}_{kRC}(v'_{kRC}) dv'_{kRC}$$

$$G_k(i_{kRL}) := \int_0^{i_{kRL}} \hat{v}_{kRL}(i'_{kRL}) di'_{kRL}.$$

Proposition 4: Arbitrary interconnections of passive capacitors with convex energy function, $\mathcal{E}_C(\mathbf{q}_C)$, voltage-controlled resistors and controlled sources, satisfy the power balance inequality

$$\int_0^t \dot{\mathbf{v}}_S^\top(\tau) \mathbf{i}_S(\tau) d\tau \geq J[\mathbf{v}_{RC}(t)] - J[\mathbf{v}_{RC}(0)] \quad (10)$$

where $J(\mathbf{v}_{RC}) := \sum_{k=1}^{n_R} J_k(v_{kRC})$. Hence, if the resistors are passive, they define passive systems with port variables $(\mathbf{i}_S, d\mathbf{v}_S/dt)$ and storage function the total resistor *co-content*. Similarly, arbitrary interconnections of passive inductors with convex energy function, $\mathcal{E}_L(\mathbf{p}_L)$, current-controlled resistors and controlled sources, satisfy the power balance inequality

$$\int_0^t \mathbf{v}_S^\top(\tau) \hat{\mathbf{i}}_S(\tau) d\tau \geq G[\mathbf{i}_{RL}(t)] - G[\mathbf{i}_{RL}(0)]$$

where $G(\mathbf{i}_{RL}) := \sum_{k=1}^{n_R} G_k(i_{kRL})$. If the resistors are passive, they define passive systems with port variables $(d\mathbf{i}_S/dt, \mathbf{v}_S)$ and storage function the total resistor *content*.

Proof: The proof of passivity of RC circuits is established as follows. First, differentiate the resistors co-content $\dot{J} = \mathbf{i}_R^\top \dot{\mathbf{v}}_R$. Then, from (8) and (9), we notice that $\mathbf{i}_C^\top \dot{\mathbf{v}}_C = \mathbf{i}_C^\top \nabla^2 \mathcal{E}_C \mathbf{i}_C \geq 0$, where the nonnegativity stems from the convexity assumption. Finally, replacing the two previous expressions in

$$\mathbf{i}_C^\top \dot{\mathbf{v}}_C + \mathbf{i}_R^\top \dot{\mathbf{v}}_R = \mathbf{i}_S^\top \dot{\mathbf{v}}_S$$

which follows from Corollary 1, and integrating from 0 to t we complete the proof.

The proof for RL circuits follows *verbatim*, but using the second identity of Corollary 1, the relation for the inductors in (9), and the definition of the content. \triangleleft

V. BRAYTON-MOSER MODEL AND GENERATION OF STORAGE FUNCTION CANDIDATES

The previous calculations show that the content and co-content functions reveal some new properties of RL and RC circuits useful for controller design, in particular identify a new passive system. Unfortunately, Tellegen's theorem alone does not seem to be enough to study RLC circuits. In this section, we will strongly rely on some fundamental results reported in [2] to generate the storage functions needed to establish similar properties for a class of RLC circuits. We recall first the following important results of [2].

Lemma 1: Consider a complete RLC circuit with the corresponding partition into subnetworks Σ_C, Σ_L . Denote with n_{RC}, n_{RL} the number of resistors in the subnetworks Σ_C and Σ_L , with port variables $(\mathbf{i}_{RC}, \mathbf{v}_{RC})$, $(\mathbf{i}_{RL}, \mathbf{v}_{RL})$, respectively. Then, there exists matrices $\mathbf{\Gamma} \in \mathbb{R}^{n_L \times n_C}$, $\mathbf{\Gamma}_C \in \mathbb{R}^{n_{RC} \times n_C}$, $\mathbf{\Gamma}_L \in \mathbb{R}^{n_{RL} \times n_L}$ with elements, $+1, -1, 0$, such that

$$\mathbf{i}_L^\top \mathbf{\Gamma} \mathbf{v}_C = \mathbf{i}_C^\top \mathbf{v}_C + \mathbf{i}_{RC}^\top \mathbf{v}_{RC} \quad (11)$$

$$\mathbf{v}_{RC} = \mathbf{\Gamma}_C \mathbf{v}_C \quad (12)$$

$$\mathbf{i}_{RL} = \mathbf{\Gamma}_L \mathbf{i}_L. \quad (13)$$

Lemma 2: The dynamics of a complete RLC circuit with regulated voltage sources in series with inductors is described by

$$\begin{aligned} \mathbf{L}(\mathbf{i}_L) \dot{\mathbf{i}}_L &= -\nabla_{\mathbf{i}_L} P + \mathbf{B}_S \mathbf{v}_S \\ \mathbf{C}(\mathbf{v}_C) \dot{\mathbf{v}}_C &= \nabla_{\mathbf{v}_C} P \end{aligned} \quad (14)$$

where

$$P(\mathbf{i}_L, \mathbf{v}_C) := \mathbf{i}_L^\top \mathbf{\Gamma} \mathbf{v}_C + G(\mathbf{\Gamma}_L \mathbf{i}_L) - J(\mathbf{\Gamma}_C \mathbf{v}_C) \quad (15)$$

is the mixed potential function and $\mathbf{B}_S \in \mathbb{R}^{n_L \times n_S}$ is a (full rank) matrix with elements +1, -1, or 0.

Remark 2: Replacing (11) and (13) in (15) we see that the mixed potential equals $\mathbf{i}_C^\top \mathbf{v}_C + \mathbf{i}_{RC}^\top \mathbf{v}_{RC} + G(\mathbf{i}_{RL}) - J(\mathbf{v}_{RC})$. The first and second right-hand side terms are the power in the capacitors and voltage-controlled resistors, respectively, and recalling Definition 2, the other right-hand side terms have also a clear interpretation in terms of power. (In particular, for linear resistors, the latter are equal to half the dissipated power). For this reason, we will say that the proposed controller design, that aims at modifying $P(\mathbf{i}_L, \mathbf{v}_C)$, is *shaping the power*.

We will now identify a subclass of these RLC circuits that satisfies the new passivity property. We find convenient to write the model in compact form as

$$\mathbf{Q}(\mathbf{i}_L, \mathbf{v}_C) \begin{bmatrix} \hat{\mathbf{i}}_L \\ \hat{\mathbf{v}}_C \end{bmatrix} = \nabla P_A \quad (16)$$

where

$$\begin{aligned} P_A(\mathbf{i}_L, \mathbf{v}_C) &:= P(\mathbf{i}_L, \mathbf{v}_C) - \mathbf{i}_L^\top \mathbf{B}_S \mathbf{v}_S \\ \mathbf{Q}(\mathbf{i}_L, \mathbf{v}_C) &:= \begin{bmatrix} -\mathbf{L}(\mathbf{i}_L) & 0 \\ 0 & \mathbf{C}(\mathbf{v}_C) \end{bmatrix} \in \mathbb{R}^{n \times n} \end{aligned} \quad (17)$$

and $n := n_L + n_C$. From (16) and noting that $\mathbf{i}_S = \mathbf{B}_S^\top \mathbf{i}_L$, we have that

$$\dot{P} = \begin{bmatrix} (\hat{\mathbf{i}}_L)^\top & \hat{\mathbf{v}}_C^\top \end{bmatrix} \mathbf{Q}(\mathbf{i}_L, \mathbf{v}_C) \begin{bmatrix} \hat{\mathbf{i}}_L \\ \hat{\mathbf{v}}_C \end{bmatrix} + \mathbf{v}_S^\top \hat{\mathbf{i}}_S. \quad (18)$$

That is, \dot{P} consists of the sum of a quadratic term plus the inner product of the sources port variables in the desired form—with the derivative of \mathbf{i}_S . Unfortunately, due to the presence of the negative sign in the first main diagonal block, $\mathbf{Q}(\mathbf{i}_L, \mathbf{v}_C)$ is sign-indefinite, and not negative definite as desired. Hence, we cannot establish a power balancing inequality from (18). Clearly, to obtain the passivity property an additional difficulty stems from the fact that $P(\mathbf{i}_L, \mathbf{v}_C)$ is also not sign definite.

To overcome these difficulties we, again, borrow inspiration from [2] and look for other suitable pairs $(\tilde{\mathbf{Q}}(\mathbf{i}_L, \mathbf{v}_C), \tilde{P}_A(\mathbf{i}_L, \mathbf{v}_C))$, which we call *admissible*, that describe the dynamics of the circuit, that is

$$\tilde{\mathbf{Q}}(\mathbf{i}_L, \mathbf{v}_C) \begin{bmatrix} \hat{\mathbf{i}}_L \\ \hat{\mathbf{v}}_C \end{bmatrix} = \nabla \tilde{P}_A. \quad (19)$$

Additional properties that we require from the admissible pairs $(\tilde{\mathbf{Q}}(\mathbf{i}_L, \mathbf{v}_C), \tilde{P}_A(\mathbf{i}_L, \mathbf{v}_C))$ are as follows.

P.1) To preserve the controlled sources variables as port variables, there should be a function $\tilde{P}(\mathbf{i}_L, \mathbf{v}_C)$, such that $\tilde{P}_A(\mathbf{i}_L, \mathbf{v}_C) = \tilde{P}(\mathbf{i}_L, \mathbf{v}_C) - \mathbf{i}_L^\top \mathbf{B}_S \mathbf{v}_S$.

P.2) To be able to establish the power balance property, we require

$$\tilde{\mathbf{Q}}(\mathbf{i}_L, \mathbf{v}_C) + \tilde{\mathbf{Q}}^\top(\mathbf{i}_L, \mathbf{v}_C) \leq 0. \quad (20)$$

P.3) Finally, to obtain passivity, $\tilde{P}(\mathbf{i}_L, \mathbf{v}_C)$ should be nonnegative.

A complete characterization of the admissible pairs $(\tilde{\mathbf{Q}}(\mathbf{i}_L, \mathbf{v}_C), \tilde{P}_A(\mathbf{i}_L, \mathbf{v}_C))$ has been reported in [8], but it requires the solution

of a partial differential equation. A more constructive procedure to generate admissible pairs is given in the following proposition which, for ease of reference, is enunciated in terms of the original RLC circuit data.⁵

Proposition 5: Consider a complete RLC circuit with regulated voltage sources in series with the inductors. Assume that the energy functions of the dynamic elements are *strictly convex*, i.e., $\nabla^2 \mathcal{E}_C, \nabla^2 \mathcal{E}_L > 0$. Then, the following hold.

i) (Sufficiency) For all $\lambda \in \mathbb{R}$, and symmetric matrix functions $\mathbf{M}(\mathbf{i}_L, \mathbf{v}_C)$, with $\mathbf{M} : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, the pair

$$\tilde{P}_A(\mathbf{i}_L, \mathbf{v}_C) := \lambda P_A + \frac{1}{2} \nabla P_A^\top \mathbf{M} \nabla P_A \quad (21)$$

$$\tilde{\mathbf{Q}}(\mathbf{i}_L, \mathbf{v}_C) := \left[\frac{1}{2} (\nabla^2 P_A) \mathbf{M} + \frac{1}{2} \nabla (\mathbf{M} \nabla P_A) + \lambda \mathbf{I} \right] \mathbf{Q} \quad (22)$$

is admissible, i.e., is such that (19) holds, with \mathbf{Q} and P_A as in (15) and (17).

ii) (Partial converse) Assume the circuit (16) admits only isolated equilibrium points. Then, given any admissible pair $(\tilde{\mathbf{Q}}, \tilde{P}_A)$ there exists λ , and \mathbf{M} such that, almost everywhere,⁶ \tilde{P}_A takes the form (21).

Proof:

i) Computing the gradient of \tilde{P}_A from (21) gives

$$\nabla \tilde{P}_A = \left[\frac{1}{2} (\nabla^2 P_A) \mathbf{M} + \frac{1}{2} \nabla (\mathbf{M} \nabla P_A) + \lambda \mathbf{I} \right] \nabla P_A.$$

Now, strict convexity of $\mathcal{E}_C, \mathcal{E}_L$ ensures the matrix \mathbf{Q} is full rank. Hence, from (22) and the aforementioned equation, we can write

$$\nabla \tilde{P}_A = \tilde{\mathbf{Q}} \mathbf{Q}^{-1} \nabla P_A = \tilde{\mathbf{Q}} \begin{bmatrix} \hat{\mathbf{i}}_L \\ \hat{\mathbf{v}}_C \end{bmatrix} \quad (23)$$

where the last identity is obtained from (16). Thus $(\tilde{\mathbf{Q}}, \tilde{P}_A)$ is admissible.

ii) Since the system (16) has only isolated equilibrium points, we have that $\nabla P_A = 0$ only at isolated points. Hence, given any function \tilde{P}_A , we can select $\lambda = 0$ and

$$\mathbf{M} = \frac{2 \tilde{P}_A}{(\nabla P_A^\top \nabla P_A)^2} \nabla P_A (\nabla P_A)^\top$$

for which (21) clearly holds (a.e.). \triangleleft

Remark 3: Some simple calculations show that a change of (state) coordinates on the dynamical system (16) acts as a similarity transformation on \mathbf{Q} . Therefore, is of no use for our purposes where we want to change the sign of \mathbf{Q} to render the quadratic form sign definite.

VI. MAIN RESULTS

In this section, we will use the background material of the previous section to establish a power balance inequality and the new passivity property for (a class of) RLC circuits. This, in its turn, will be applied to stabilize an equilibrium via power shaping.

Theorem 1 (Power Balance Inequality and new Passivity Property): Consider a complete RLC circuit with regulated voltage sources in series with inductors. Assume the following.

- A.1) The inductors and capacitors are passive and have *strictly convex* energy functions.
- A.2) The voltage controlled resistors are linear, that is, $\mathbf{i}_{RC} = \mathbf{R}_C^{-1} \mathbf{v}_{RC}$, with $\mathbf{R}_C = \text{diag}\{R_{kC}\} > 0$.

⁵To simplify the notation, in the sequel we omit the arguments of the functions, writing them explicitly only when the function is first defined.

⁶As shown in the proof, the qualifier (a.e.) stands for the existence of possible singular points. These points can be avoided with standard regularization procedures, but is omitted here for brevity.

A.3) Uniformly in $\mathbf{i}_L, \mathbf{v}_C$, we have⁷

$$\left\| \mathbf{C}^{\frac{1}{2}}(\mathbf{v}_C) \tilde{\mathbf{R}}_C \Gamma^\top \mathbf{L}^{-\frac{1}{2}}(\mathbf{i}_L) \right\| \leq 1 - \epsilon$$

for some $\epsilon > 0$, where $\tilde{\mathbf{R}}_C^{-1} := \Gamma_C^\top \mathbf{R}_C^{-1} \Gamma_C$ is a full rank matrix, and $\|\cdot\|$ is the spectral norm of a matrix.

Under these conditions, we have the *power balance inequality*

$$\int_0^t \mathbf{v}_S^\top(\tau) \hat{\mathbf{i}}_S(\tau) d\tau \geq \tilde{P}[\mathbf{i}_L(t), \mathbf{v}_C(t)] - \tilde{P}[\mathbf{i}_L(0), \mathbf{v}_C(0)] \quad (24)$$

where

$$\begin{aligned} \tilde{P}(\mathbf{i}_L, \mathbf{v}_C) = & \frac{1}{2} \left(\Gamma^\top \mathbf{i}_L - \tilde{\mathbf{R}}_C^{-1} \mathbf{v}_C \right)^\top \tilde{\mathbf{R}}_C \left(\Gamma^\top \mathbf{i}_L - \tilde{\mathbf{R}}_C^{-1} \mathbf{v}_C \right) \\ & + \frac{1}{2} \mathbf{i}_L^\top \Gamma \tilde{\mathbf{R}}_C \Gamma^\top \mathbf{i}_L + G. \end{aligned} \quad (25)$$

Furthermore, if

A.4) the current controlled resistors are *passive*

then the circuit defines a *passive* system with port variables $(\mathbf{v}_S, \hat{\mathbf{i}}_S)$ and storage function $\tilde{P}(\mathbf{i}_L, \mathbf{v}_C)$.

Proof: The proof consists of defining the parameters \mathbf{M} and λ of Proposition 5 so that, under the conditions A.1)–A.4) of the theorem, the resulting pair $(\mathbf{Q}, \tilde{P}_d)$ verifies the properties P.1)–P.3).

First, notice that under Assumption A.2) the co-content takes the form $J(\mathbf{v}_C) = (1/2) \mathbf{v}_C^\top \tilde{\mathbf{R}}_C^{-1} \mathbf{v}_C$. To ensure that \tilde{P}_d is linear in \mathbf{v}_S , as required in P.1), we see from (21) that we can select

$$\mathbf{M} = \begin{bmatrix} 0 & 0 \\ 0 & 2\tilde{\mathbf{R}}_C \end{bmatrix}, \quad \lambda = 1. \quad (26)$$

For which, after some simple calculations with (14) and (22), we get

$$\tilde{\mathbf{Q}} = \begin{bmatrix} -\mathbf{L} & 2\tilde{\mathbf{R}}_C \Gamma \mathbf{C} \\ 0 & -\mathbf{C} \end{bmatrix}. \quad (27)$$

Assumption A.1) ensures that \mathbf{C} and \mathbf{L} are positive definite. A Schur complement analysis reveals that, under Assumption A.3), (20) of P.2) holds. This proves the power balance inequality.

To establish P.3), we replace (26) and (15) in (21), and complete a square to show that \tilde{P} takes the form (25). The first and second right hand terms are positive because of positivity of $\tilde{\mathbf{R}}_C$, [Assumption A.2)], and the content G is also nonnegative in view of Assumption A.4. This completes the proof. \square

Remark 4: Assumption A.3) is satisfied if the voltage controlled resistances R_{kC} are “small.” Recalling that these resistors are in parallel with the capacitors, this means that the *coupling between inductors and capacitors is weak*—with the capacitors short-circuited in the limiting case $R_{kC} = 0$.

The theorem below proves that complete RLC circuits with strictly convex energy function and linear voltage controlled resistors are stabilizable via power-shaping—without requiring Assumptions A.3) or A.4)—but only provided that the number of control signals is “sufficiently large” to shape the mixed potential function and add the damping.

Theorem 2 (Stabilization via Power Shaping): Consider a complete RLC circuit satisfying Assumptions A.1) and A.2) of Theorem 1, and a desired (admissible) equilibrium $(\mathbf{i}_L^*, \mathbf{v}_C^*) \in \mathbb{R}^n$. Assume there exists a function $P_a : \mathbb{R}^n \rightarrow \mathbb{R}$ verifying the following.

A.5) (Realizability) $\mathbf{B}_S^\top \nabla P_a = 0$, where $\mathbf{B}_S^\top \mathbf{B}_S = 0$.

A.6) (Equilibrium assignment) $\nabla P_a(\mathbf{i}_L^*) + \nabla \mathbf{i}_L G(\Gamma_L \mathbf{i}_L^*) + \Gamma \tilde{\mathbf{R}}_C \Gamma^\top \mathbf{i}_L^* = 0$.

⁷As discussed in Remark 4, this constraint is satisfied if the electromagnetic coupling is sufficiently “weak.”

A.7) (Damping injection) Uniformly in \mathbf{i}_L , $\nabla^2 P_a + \nabla_{\mathbf{i}_L}^2 G \geq R_a \mathbf{I}$, for some sufficiently large $R_a > 0$.

Under these conditions, the circuit is stabilizable via *power shaping*. More precisely, the control law

$$\mathbf{v}_S = - \left(\mathbf{B}_S^\top \mathbf{B}_S \right)^{-1} \mathbf{B}_S^\top \nabla P_a \quad (28)$$

ensures that all bounded trajectories satisfy $\lim_{t \rightarrow \infty} (\mathbf{i}_L(t), \mathbf{v}_C(t)) = (\mathbf{i}_L^*, \mathbf{v}_C^*)$. Furthermore, if the characteristic functions of the dynamic elements are such that $(\mathbf{p}_L, \mathbf{q}_C) = (\hat{\mathbf{p}}_L(\mathbf{i}_L), \hat{\mathbf{q}}_C(\mathbf{v}_C))$ is a global diffeomorphism then all trajectories are bounded and the equilibrium is *globally attractive*.

Proof: From Lemma 2, we know that the circuit dynamics is described by (14) and (15). Now, under Assumption A.5), the control law (28) satisfies $\mathbf{B}_S \mathbf{v}_S = -\nabla P_a$. This leads to the closed-loop dynamics $\mathbf{Q} \begin{bmatrix} \dot{\mathbf{i}}_L \\ \dot{\mathbf{v}}_C \end{bmatrix} = \nabla P_d$, where $P_d(\mathbf{i}_L, \mathbf{v}_C) := P + P_a$. From Assumption A.1), we have that \mathbf{Q} is full rank and consequently the equilibria are the extrema of P_d . Now, from (15) and Assumption A.2) we have that

$$\nabla P_d = \begin{bmatrix} \Gamma \mathbf{v}_C + \nabla \mathbf{i}_L G + \nabla P_a \\ \Gamma^\top \mathbf{i}_L - \tilde{\mathbf{R}}_C^{-1} \mathbf{v}_C \end{bmatrix}.$$

Since all admissible equilibria satisfy $\mathbf{v}_C^* = \tilde{\mathbf{R}}_C \Gamma^\top \mathbf{i}_L^*$, we clearly have that $\nabla_{\mathbf{v}_C} P_d(\mathbf{i}_L^*, \mathbf{v}_C^*) = 0$. On the other hand, Assumption A.2) and A.7) ensure that the function $P_a(\mathbf{i}_L) + G(\Gamma_L \mathbf{i}_L) + (1/2) \mathbf{i}_L^\top \Gamma \tilde{\mathbf{R}}_C \Gamma^\top \mathbf{i}_L$ is strongly convex, and consequently that it has a unique global minimum at the point where its gradient is zero. This, together with Assumption A.6), ensures $(\mathbf{i}_L^*, \mathbf{v}_C^*)$ is the unique equilibrium of the closed-loop system.

Once we have achieved the power shaping we will now apply Proposition 5 to generate another admissible pair $(\mathbf{Q}, \tilde{P}_d)$ with $\mathbf{Q} + \mathbf{Q}^\top < 0$ —notice the strict inequality. We make at this point the important observation that, since $\nabla \tilde{P}_d = \tilde{\mathbf{Q}} \mathbf{Q}^{-1} \nabla P_d$ (which follows from (23)), the extrema of all new mixed potentials \tilde{P}_d will coincide with the extrema of P_d .

We apply the transformations of Proposition 5 to the closed-loop system above with the parameters $\lambda = -1$, $\mathbf{M} = \begin{bmatrix} (2/R_a) \mathbf{I} & 0 \\ 0 & 0 \end{bmatrix}$, that yields

$$\tilde{\mathbf{Q}} = \begin{bmatrix} \left[-\frac{2}{R_a} (\nabla^2 P_a + \nabla_{\mathbf{i}_L}^2 G) - \mathbf{I} \right] \mathbf{L} & 0 \\ -\frac{2}{R_a} \Gamma^\top \mathbf{L} & -\mathbf{C} \end{bmatrix}$$

whose symmetric part is negative definite for sufficiently large R_a . Consequently, along the closed-loop dynamics, which can also be described by $\tilde{\mathbf{Q}} \begin{bmatrix} \dot{\mathbf{i}}_L \\ \dot{\mathbf{v}}_C \end{bmatrix} = \nabla \tilde{P}_d$, we have

$$\dot{\tilde{P}}_d = \frac{1}{2} \nabla \tilde{P}_d^\top \tilde{\mathbf{Q}}^{-\top} (\tilde{\mathbf{Q}} + \tilde{\mathbf{Q}}^\top) \tilde{\mathbf{Q}}^{-1} \nabla \tilde{P}_d \leq -\alpha |\nabla \tilde{P}_d|^2$$

for some $\alpha > 0$, where $|\cdot|$ is the Euclidian norm. Convergence of all bounded trajectories follows immediately from LaSalle's invariance principle and the fact that $|\nabla \tilde{P}_d| = 0$ only at the desired equilibrium.⁸

To prove boundedness of trajectories we apply the change of coordinates $(\mathbf{p}_L, \mathbf{q}_C) = (\hat{\mathbf{p}}_L(\mathbf{i}_L), \hat{\mathbf{q}}_C(\mathbf{v}_C))$ to the closed-loop system to obtain

$$\begin{bmatrix} \dot{\mathbf{p}}_L \\ \dot{\mathbf{q}}_C \end{bmatrix} = \begin{bmatrix} 0 & -\Gamma \\ \Gamma^\top & -\tilde{\mathbf{R}}_C^{-1} \end{bmatrix} \begin{bmatrix} \nabla \mathcal{E}_L \\ \nabla \mathcal{E}_C \end{bmatrix} - \begin{bmatrix} \nabla_{\mathbf{i}_L} G \left(\Gamma_L \hat{\mathbf{i}}_L(\mathbf{p}_L) \right) + \nabla P_a \left(\hat{\mathbf{i}}_L(\mathbf{p}_L) \right) \\ 0 \end{bmatrix}$$

where we have denoted the inverse function $\mathbf{i}_L := \hat{\mathbf{i}}_L(\mathbf{p}_L)$ and recalled from (9) that $\mathbf{v}_C = \nabla \mathcal{E}_C$ and $\mathbf{i}_L = \nabla \mathcal{E}_L$.

⁸The explicit expression of \tilde{P}_d is of no interest for our derivations, as LaSalle's invariance principle imposes no particular positivity constraint on this function.

From Assumption A.1), we have that the total energy, $\mathcal{E} = \mathcal{E}_C + \mathcal{E}_L$, is a *positive radially unbounded* function. Evaluating its time derivative, we get

$$\dot{\mathcal{E}} = -\nabla^\top \mathcal{E}_C \dot{\mathbf{R}}_C^{-1} \nabla \mathcal{E}_C - \nabla \mathcal{E}_L^\top \left[\nabla_{\mathbf{i}_L} G(\mathbf{T}_L \dot{\mathbf{i}}_L(\mathbf{p}_L)) + \nabla P_a(\dot{\mathbf{i}}_L(\mathbf{p}_L)) \right]. \quad (29)$$

Assumption A.7) states that the function $G(\mathbf{T}_L \dot{\mathbf{i}}_L) + P_a(\dot{\mathbf{i}}_L)$ is *strongly convex*. The latter ensures that the second right-hand side term in (29) is positive outside some ball $|\dot{\mathbf{i}}_L| = b$, and consequently $\dot{\mathcal{E}}$ is negative outside a compact set. This proves global boundedness of the solutions and completes the proof. \triangleleft

Remark 5: Clearly, all assumptions of Theorem 2 are constraints related with the “degree of under-actuation” of the circuit. All conditions are obviated in the extreme case where $\mathbf{B}_S = \mathbf{I}$ when we can add an arbitrary power function P_a . Also, the rather restrictive Assumption A.3) of Theorem 1 is conspicuous by its absence—this means that we *do not* assume that the circuit to be controlled is already passive.

VII. CONCLUSION AND OUTLOOK

Our main motivation in this note was to propose an alternative to the well-known method of energy shaping stabilization of physical systems—in particular, to the physically appealing technique of energy balancing (also known as control by interconnection for dynamic controllers) which as pointed out in [11] and [14] is severely stymied by the existence of pervasive damping. In this note, we have, for nonlinear RLC circuits, put forth the paradigm of power shaping and shown that it is not restricted to systems without pervasive dissipation.

The starting point for the formulation of the power shaping idea are some new power balancing and passivity properties established for a class of nonlinear RLC circuits with convex energy function and weak electromagnetic coupling. To enlarge the class of circuits that enjoy these properties we have made extensive use of Proposition 5 which provides a procedure to generate alternative circuit topologies that reveal, through the new admissible pairs $(\dot{\mathbf{Q}}, \dot{\mathbf{P}})$, properties of the original circuit that we can exploit in our controller design.

The following open issues are currently under investigation.

- Instrumental for our developments is the exploitation of a geometrical property of RLC circuits, namely that voltages and currents live in orthogonal spaces, i.e., Tellegen’s theorem. Dirac structures, as proposed in [14], provide a natural generalization to this theorem, characterizing in an elegant geometrical language the key notion of power preserving interconnections. It seems that this is the right notion to try to extend our results beyond the realm of RLC circuits, e.g., to mechanical or electromechanical systems. (A related question is whether we can find Brayton–Moser like models for this class of systems; see [1] and [5]).
- In this note, we have elaborated only on overcoming the dissipation obstacle of energy balancing, but it has also been mentioned that power shaping naturally allows the addition of (approximate) derivative actions in the control to enhance the transient response. Indeed, following the procedure of [10, Sec. 3.2] it is possible to show that we can add to the controller (28) an approximate differentiation term $\text{diag}\{(-k_{Di}s)/(\tau_i s + 1)\} \dot{\mathbf{i}}_L$, with $k_{Di}, \tau_i > 0$, preserving the same stability properties of Theorem 2. The theoretical and practical implications of adding derivative actions in power shaping is currently under investigation.
- We have considered here only voltage sources which suggest that current sources can be treated analogously using an alternative definition of the mixed potential.

- Parallel to the developments reported in this note we are investigating linear RLC circuits. In this case there is a clear interpretation, in terms of the phase of the driving point impedance, of the circuits that satisfy the new passivity property. Furthermore, using some well known relationships between the impedance and the average stored energy, e.g., [3, eq. (5.6), Ch. 9], we can fully characterize these circuits in terms of their energy functions. The outcome of this research will be reported elsewhere.
- The expression $\dot{\mathbf{v}}_S^\top(t) \dot{\mathbf{i}}_S(t)$ (or $\mathbf{v}_S^\top(t) \dot{\mathbf{i}}_S(t)$) has a direct relationship with the notion of *reactive power*, as classically defined for linear circuits. Indeed, if we take the average of this signal on a period and expand in Fourier series, the first component coincides with the standard definition of reactive power for a two terminal circuit with sinusoidal voltage. Adopting this new “definition” of reactive power for nonlinear circuits might prove instrumental to formally study problems of reactive power compensation—an area of intense research activity in power electronics.

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